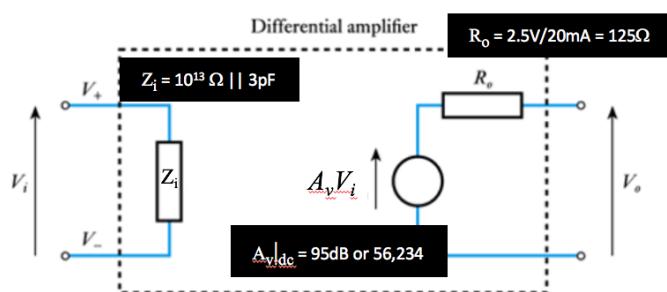


**Problem Sheet 1 Solutions**  
(Operation Amplifiers – Lectures 1 – 3)

1. a) This tests your ability to read and interpret datasheets. Extracted from datasheet:

Common Mode Input Impedance	$Z_{CM}$	—	$10^{13} \text{ II}_6$	—	$\Omega \text{ IIpF}$
Differential Input Impedance	$Z_{DIFF}$	—	$10^{13} \text{ II}_3$	—	$\Omega \text{ IIpF}$
DC Open Loop Gain	$A_{OL}$	95	110	—	dB
Output Short Circuit Current	$I_{SC}$	—	20	—	mA

There is no explicit specification for output resistance. However, one can derive this approximately from the short circuit current.



b) From datasheet:

Linear Region Maximum Output Voltage Swing	$V_{OUT}$	$V_{SS} + 0.050$	—	$V_{DD} - 0.050$	V	$R_L = 25\text{k}\Omega$ to $V_{DD}/2$ , $A_{OL} \geq 100\text{dB}$
	$V_{OUT}$	$V_{SS} + 0.100$	—	$V_{DD} - 0.100$	V	$R_L = 5\text{k}\Omega$ to $V_{DD}/2$ , $A_{OL} \geq 95\text{dB}$

The specification does not provide comprehensive answer to this question. However, the datasheet information suggest two thing important things:

- 1) If the output load is high (25k) i.e. output current is low, output range can be  $\pm 50\text{mV}$  from the power supply voltages
- 2) This output range is dependent on the output current. As  $R_L$  is reduced to 5k, the output range is reduced because the so-call headroom is now increased to  $\pm 100\text{mV}$ .

Figure 2-20 of the datasheet provides more detailed specification up to 10mA output current.

c) From datasheet and Figure 2.1, we see that the Gain-bandwidth product is typically 2.8MHz.

Therefore, if maximum signal frequency is 100kHz, the maximum gain would theoretically be x28 or lower. This is particularly true because the GBP value of 2.8MHz is not worst case!

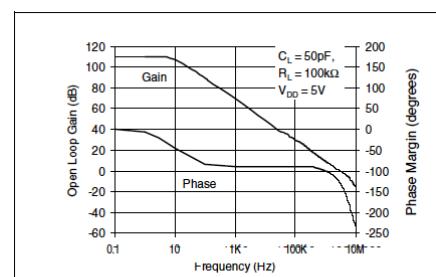
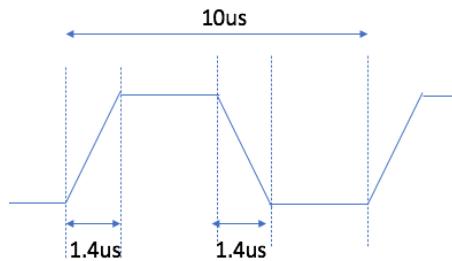


FIGURE 2-1: Open Loop Gain, Phase Margin vs. Frequency

d) This relates to the slew rate limit of the op-amp.

$$\text{Slew Rate} \quad | \quad \text{SR} \quad | \quad - \quad | \quad 2.3 \quad | \quad - \quad | \quad \text{V}/\mu\text{s} \quad | \quad G = +1\text{V/V}, V_{DD} = 5\text{V}$$

Therefore this typically takes  $3.3/2.3 \text{ us} = 1.4\text{us}$  to go from 0V to 3.3V or 3.3V to 0V. Hence the waveform would look like:



2. The gain of the system from  $V_{in}$  to  $V_{out}$  is:

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R1}{Rs}$$

However, the circuit is used in a unusual way in that  $V_{in}$  is constant, instead it is the gain itself that is changed by the lorry affecting the feedback resistor value.

Since  $V_{in} = 1$ ,

$$V_{out} = 1 + \frac{R1}{R_0 + \alpha W}$$

Therefore

$$\frac{dV_{out}}{dW} = -R1\alpha/(R_0 + \alpha W)^2$$

3. We normally assume that an op-amp has near infinite input impedance. This question demonstrate what if this assumption does not apply, and the input impedance is in fact  $R_p$ ?

For  $A_V = \infty$ ,  $V+ = V-$ . Since X is the virtual earth point ( $V_x = 0$ ), no current flows in  $R_p$ .  $V_{out}$  is therefore not affected by  $R_p$  at all.

$$V_{out} = -R_f \left( \frac{V_1}{R_2} + \frac{V_2}{R_1} \right).$$

For  $A_V < \infty$ , we need to include the effects of  $R_p$  in the calculation.

Apply KCL at  $V_x$ :

$$\frac{V_x}{R_p} = \left( \frac{V_1 - V_x}{R_2} + \frac{V_2 - V_x}{R_1} + \frac{V_{out} - V_x}{R_f} \right)$$

We also know that:

$$V_{out} = -A_V V_x$$

Eliminate  $V_x$  from the first equation and we have an accurate but rather tedious equation:

$$V_{out} = -\left( \frac{V_1}{R_2} + \frac{V_2}{R_1} \right) [R_f || A_V (R_1 || R_2 || R_p || R_f)]$$

If  $R_p \gg$  all other resistances in the equation and  $A_V$  is large,

$$[R_f || A_V (R_1 || R_2 || R_p || R_f)] \approx R_f$$

and we have the familiar voltage summing equation.

4. Real op-amps have another imperfection known as input offset voltage  $V_{OS}$ . For example MCP601 has a worst case  $V_{OS}$  of  $\pm 3\text{mV}$  at industrial temperature range.

Use the principle of superposition and consider  $V_{in}$  and  $V_{OS}$  separately with the other voltage source set to 0, and calculate  $V_{out}$  in turn gives:

Assume  $V_{OS} = 0$ ,

$$V_{out1} = V_{in}(1 + \frac{R_1}{R_2})$$

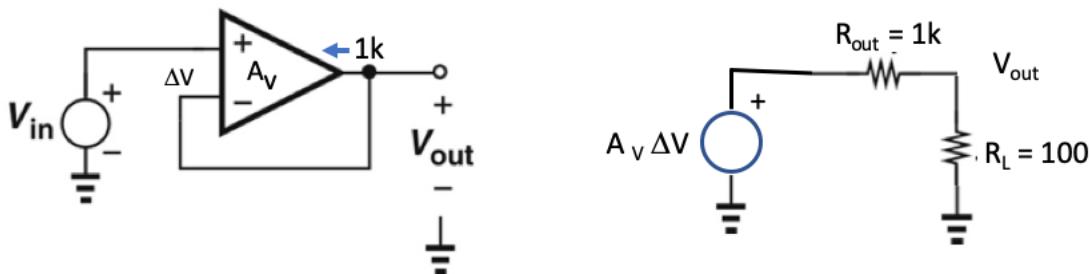
Assume  $V_{in} = 0$ , voltage across  $R_2$  is  $V_{OS}$ . Therefore, apply voltage divider principle,

$$V_{OS} = V_{out2}(\frac{R_2}{R_1+R_2})$$

Combine the two gives:

$$V_{out} = (V_{in} + V_{OS})(1 + \frac{R_1}{R_2})$$

5. This question demonstrates how negative feedback reduces the output impedance of the amplifier by trading it off for the open-loop gain. The op-amp would give a gain of 1000, and yet, we only demand it to give a gain of 1. Negative feedback allows us to trade this for improved effective output impedance. Here is how it works.



The micromodel of the output circuit is simply a voltage source  $= A_v (V_{in} - V_{out})$  driving a resistor divider with a load  $R_L$  of 100 ohm and an output resistance  $R_o$  of 1k ohm.

Therefore:

$$V_{out} = A_v(V_{in} - V_{out}) \frac{R_L}{R_L + R_o}$$

Since  $(1 + x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{R_L + R_o}{A_v R_L}} \approx 1 - \frac{R_o + R_L}{A_v R_L} = 1 - \text{error (for } A_v \gg 1)$$

Now we know the error, which is the second term here is within 0.5%. Therefore:

$$\left| \frac{R_o + R_L}{A_v R_L} \right| = 0.005$$

$$A_v = \left| \frac{R_o + R_L}{0.005 R_L} \right| = \frac{1000 + 100}{0.005 * 100} = 2200$$

6. Ideal gain  $(1+R_1/R_2) = 4$ , and  $R_1+R_2 = 20k$   
 Hence  $R_1 = 15k$  and  $R_2 = 5k$ .

Unfortunately the gain is not exactly 4 due to the finite Av.  
 Instead, the gain is:

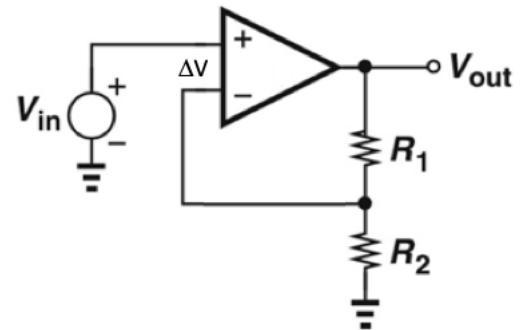
$$\frac{V_{out}}{V_{in}} = \frac{A_v}{1 + A_v \frac{R_2}{R_2 + R_1}}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_v}{1 + A_v \frac{R_2}{R_2 + R_1}} \approx (1 + \frac{R_1}{R_2})[1 - (1 + \frac{R_1}{R_2}) \frac{1}{A_v}]$$

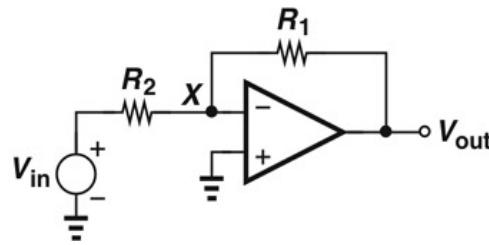
Since we know  $(1 + \frac{R_1}{R_2}) = 4$ , the nominal gain, and the gain error is 0.2%,

$$(1 + \frac{R_1}{R_2}) \frac{1}{A_v} = 0.002$$

Hence,  $Av = 4/0.002 = 2000$ .



7. Nominal gain =  $R_1/R_2 = -8$ , Gain error =  $\pm 10\%$ .



$R_1$  and  $R_2$  needs to be at least accurate to  $\pm 5\%$ . Check:  $R_1 \rightarrow +5\%$ ,  $R_2 \rightarrow -5\%$ , Absolute gain error =  $+10.5\%$  (from 8).  $R_1 \rightarrow -5\%$ ,  $R_2 \rightarrow 5\%$ ,  $-9.5\%$ .

Assume that  $A_v$  is high enough for the equation  $G = -R_2/R_1$  applies. Now assume 0.5% is available as gain error due to  $A_v$  being finite. What is the minimum value of  $A_v$ ?

$$V_x = -V_{out}/A_v$$

$$\frac{V_{in} - V_x}{R_2} = \frac{V_x - V_{out}}{R_1}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{R_2} \frac{1}{1 + \frac{1}{A_v} \left(1 + \frac{R_1}{R_2}\right)} \simeq -\frac{R_1}{R_2} \left[1 + \frac{1}{A_v} \left(1 + \frac{R_1}{R_2}\right)\right]$$

$$|error| < 0.5\% = \left| \frac{1}{A_v} \left(1 + \frac{R_1}{R_2}\right) \right| \simeq \frac{1}{A_v} (1 + 8)$$

$A_v > 1800$ .